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MATHEMATICS.—*A straight line chart for the solution of spherical triangles.* F. E. WRIGHT, Geophysical Laboratory.

A straight line chart for the solution of a spherical triangle, in which two sides and the included angle are given, has been in use for many years. In 1891 M. d'Ocagne[1] published a straight line chart for the solution of the equation for the spherical distance between two points on a sphere whose latitudes and longitudes are known

$$2\cos c = (1 + \cos L)\cdot\cos(a - b) - (1 - \cos L)\cdot\cos(a + b)$$

in which c is the spherical distance, L, the difference in longitude, and a and b are the latitudes. This chart was reproduced in 1919 by Peddle[2] who derived it by the general method of determinants. In 1917 Littlehales[3] derived, independently and by still another method, the same chart for the solution of the same equation expressed, however, in haversines

$$(\text{hav } a = \sin^2\frac{a}{2} = \frac{1 + \cos a}{2})$$

and of the form

$$\text{hav } z = \text{hav}(CoL - PD) + \text{hav}(CoL + PD) - \text{hav}(CoL - PD).\ \text{hav } t$$

in which z is the zenith distance, CoL, the colatitude, PD, the polar distance, and t, the hour angle. The Littlehales' chart is published

[1] Nomographie. Les calculs usuels effectués au moyen des abaques. Essai d'une théorie générale. Paris. 1891; Traité de nomographie. Paris. 327. 1899. (abaque de la distance sphérique). See also M. Collignon, Note sur la détermination de l'heure du passage du Soleil dans un plan vertical. Journ. de l'École Polytechnique, sér. 2, **4**: 123. 1898.

[2] J. B. Peddle, The construction of graphical charts. New York. 149–151. 1919.

[3] G. W. Littlehales, Altitude, azimuth, and hour angle diagram. Proc. U. S. Naval Institute **43**: 2541–2546. 1917.

by the Hydrographic Office of the U. S. Navy[4] as a separate sheet measuring 56.8 cm. (22.4 inches) on a side, and serves exceedingly well the purpose for which it is intended.

It has, however, apparently not been realized heretofore that this chart, properly labeled, is competent to solve not only the case of an oblique spherical triangle in which two sides and the included angle are given and the third side is sought, but also any spherical triangle for which any three of its six angles are given and the other three are sought. To show how this can be done and to direct attention to the usefulness of this chart in problems of spherical trigonometry is the purpose of this note.

The three fundamental equations of spherical trigonometry are

$$\sin a : \sin b = \sin A : \sin B \qquad \text{(sine formula)} \quad (1)$$

$$\left.\begin{aligned} \cos a &= \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A \quad (2) \\ \cos A &= -\cos B \cdot \cos C + \sin B \cdot \sin C \cdot \cos a \quad (3) \end{aligned}\right\} \text{(cosine formulas)}$$

in which a, b, c are the sides, and A, B, C are the solid angles opposite a, b, c respectively, of the spherical triangle. Other equations of similar form are derived from these equations by cyclical transposition of the letters in the usual manner.

The cosine formulas can be rewritten in the form[5]

$$2\cos a = [\cos(b-c) - \cos(b+c)]\cdot(1+\cos A) + 2\cos(b+c) \quad (2a)$$

$$2\cos A = [\cos(B-C) - \cos(B+C)]\cdot(1+\cos a) - 2\cos(B-C) \quad (3a)$$

which contains only cosine functions and from which the cosine chart is easily derived. In equation (2a) let $\cos a = y$, $1 + \cos A = x$, $\cos(b-c) - \cos(b+c) = m$, and $\cos(b+c) = n$; the equation reduces then to the form

$$y = \frac{m}{2}\cdot x + n$$

which is that of a straight line in rectangular coördinates. The line passes through the points: $x = 0$, $y = n = \cos(b+c)$; and $x = 2$, $y = m + n = \cos(b-c)$. To construct the chart let O be the origin of coördinates; draw on the zero ordinate a cosine scale for the values

[4] Sheet no. 2776. Published June, 1917, at Washington, D. C. Plate 1 which accompanies this paper is a reproduction, on a reduced scale, of this chart. The scales, however, have been relabeled and other scales added. It is a pleasure for the writer to express herewith his indebtedness to Dr. G. W. Littlehales of the U. S. Hydrographic Office for permission thus to reproduce the chart.

[5] Expressed in haversines the cosine equations read

$$\text{hav } a = \text{hav}(b+c) + \text{hav}(b-c) - \text{hav}(b+c)\cdot\text{hav } A$$

$$\text{hav } A = \text{hav}(B+C) + \text{hav}(B-C) - \text{hav}(B+C)\cdot\text{hav } a$$

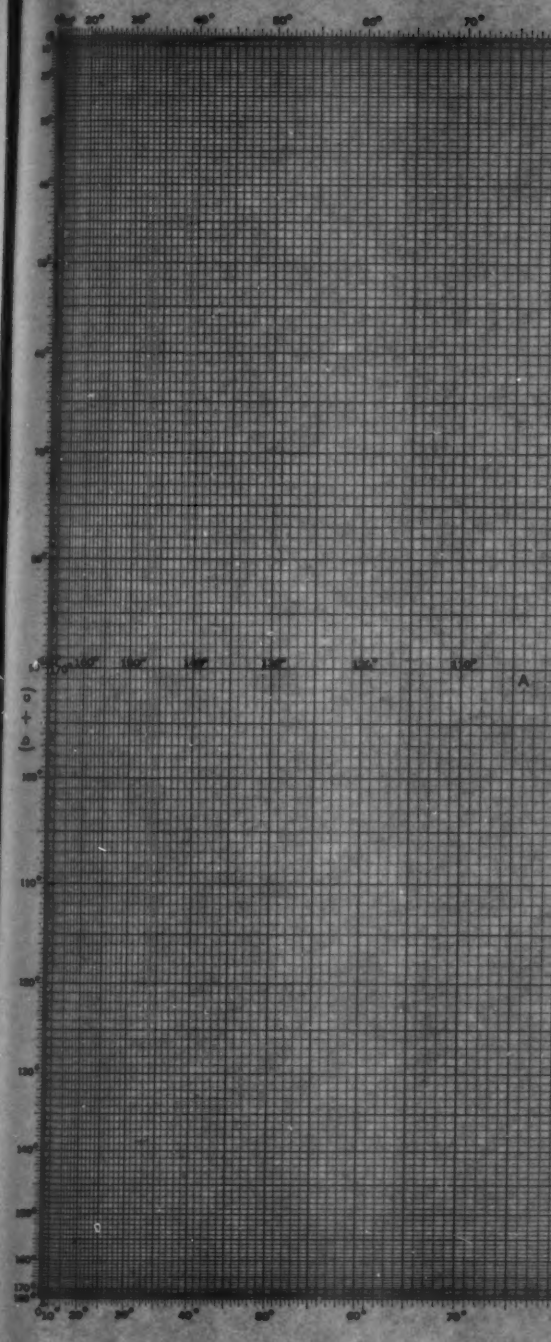
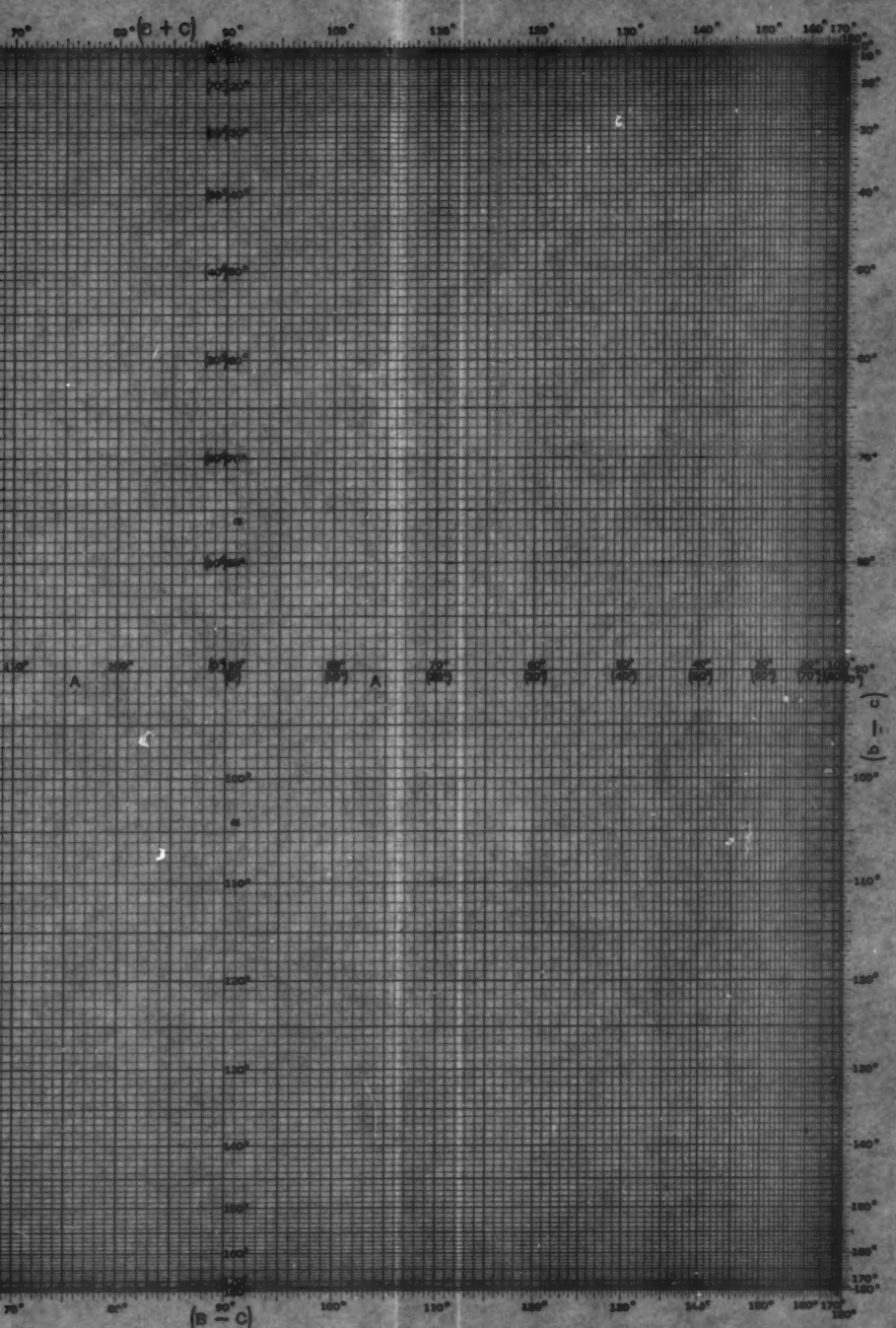


Plate 1. On this plate, which is a reproduction of sheet N
scales of figs. 3 and 4 have been combined into one diagram, w
of solving any spherical triangle in which any three of th
construction depends on the elements given. For the six poss



n of sheet No. 2776 of the U. S. Hydrographic Office but with relabeled and additional scales, the
e diagram, which is made to serve both as a cosine chart and as a sine chart, thus rendering it capable
three of the six elements are given; sought are the three remaining elements. The mode of
the six possible cases the constructions are described in the text and also under figs. 1 to 4.

$\cos(b + c)$; draw on the ordinate, NQ, at $x = 2$, a cosine scale to represent the values $\cos\ (b - c)$; along the x-axis, OQ, draw a cosine scale to represent the values $(1 + \cos A)$, as indicated in figure 1. To solve a spherical triangle in which b, c, and A are known, pass a straight line through the points L and N, for which $LO = \cos(b + c)$ and $NQ = \cos(b - c)$. For the point P on this line $PR = \cos a = y$, $OR = 1 + \cos A = x$ and from the similarity of the triangles NML and PSL we have

$$\frac{NM}{ML} = \frac{PS}{LS} \text{ or } \frac{\cos(b - c) - \cos(b + c)}{2} = \frac{\cos a - \cos(b + c)}{1 + \cos A}$$

which is identical with equation (2a). To solve equation (2a) by means of the chart (plate 1) pass a straight line through the points $(b + c)$ on the left side of the chart and $(b - c)$ on the right hand side and find the ordinate $PR = \cos a$ (labeled a) which passes through the abscissa $OR = 1 + \cos A$ and is labeled simply A.

For equation (3a) the mode of construction is similar to that for (2a) except for the signs of the angles. If, in equation (3a), $\cos A = y$, $1 + \cos a = x$, $\cos (B - C) - \cos (B + C) = m$, and $\cos (B - C) = n$, the equation reduces to the form

$$y = \frac{m}{2} x - n$$

which represents a straight line passing through the points $x = O$, $y = -n = -\cos(B - C)$ and $x = 2$, $y = -\cos(B + C)$. The graphical solution is illustrated in Fig. 2, in which $DT = \cos(B - C)$ $HG = \cos(B + C)$, $TK = 1 + \cos a$ and $KP = -\cos A = \cos (180° - A)$. In the similar triangles DEP and DFG we have

$$\frac{DE}{DF} = \frac{EP}{FG} \text{ or } \frac{1 + \cos a}{2} = \frac{\cos(B - C) - \cos A}{\cos(B - C) - \cos(B + C)}$$

This expression expanded is identical with (3a) except that the sign of $\cos A$ is negative, which means that in reading A its supplement is to be taken; in other words the A scale is inverted, as indicated on the ordinate scale through the center of fig. 2. Except for this change in sign the two charts are identical and the single cosine chart can be used for the solution of the two equations (2a) and (3a); but in the first case the ordinate is $\cos a$, and in the second the ordinate is $\cos A$. This is an undesirable feature which can be readily eliminated by rotating the x, y axes of the chart for (3a) through 90°;

this means an interchange of the x, y axes and is done by suitable designation of the angles on the chart. This change is important and greatly extends the usefulness of the chart because the intersection of the two straight lines, represented by equations (2a) and (3a), determines both a and A simultaneously. The exact labeling of the several cosine scales is shown in fig. 3 and on plate 1, and once accomplished needs no further attention.

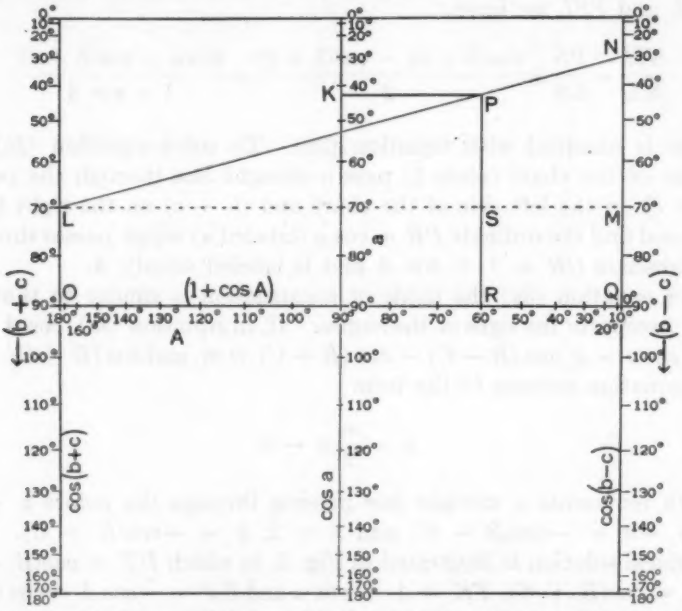


Fig. 1. Straight line cosine diagram for the graphical solution of a spherical triangle in which two sides b, c and an included angle A or the third side a are given; to find the third side a or the included angle A. Pass a straight line through the points L [angle $(b+c)$; distance $LO = \cos(b+c)$] and N [angle $(b-c)$; distance $NQ = \cos(b-c)$]; intersection P of this line with the ordinate erected at R [angle A; distance $OR = 1 + \cos A$; or angle a; distance $PR = \cos a$] is the desired point P which determines A and a.

The use of the chart is best illustrated by a series of examples which cover all possible cases that may arise. To solve these examples, however, the sine formula (1) is required and for its solution the cosine chart suffices, provided the sine scales are properly indicated thereon. If, in the sine formula $\sin a = y_1$, $\sin b = y$, $\sin A = x_1$, and $\sin B = x$, we have $y = \frac{y_1}{x_1} \cdot x$, which is the equation of a straight line passing through the origin. If therefore on plate 1, the ori-

gin be considered to be shifted from O to the center of the chart and the sines of the angles are plotted in both the x and y directions (fig. 4), the lines of plate 1 suffice, providing the numbers in parentheses are used for the angles, as indicated in figure 4 and also on plate 1 (*NE* quadrant). When the scale numbers in parentheses are used the chart is a sine chart and is so designated in the examples; otherwise it is called the cosine chart.

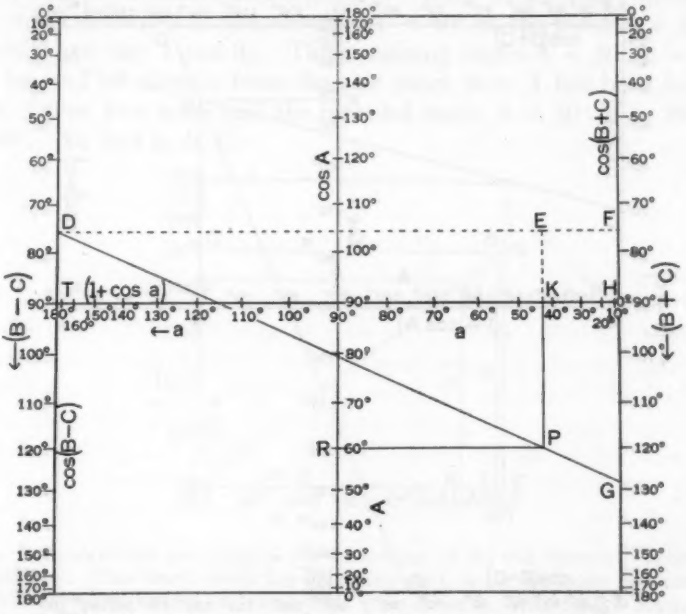


Fig. 2. Straight line cosine diagram for the solution of a spherical triangle in which two angles B, C and one included side a or the third angle A are given; sought either the third angle A, or the included side a. Construction similar to that of fig. 1. Intersection of line through points D [$TD = \cos(B-C)$] and G [$GH = \cos(B+C)$] with the ordinate KP erected at K ($TK = 1 + \cos a$, $KP = -\cos A$), determines the point P and with it A and a.

(1) Given the three sides of a spherical triangle: $a = 42°42'$ $b = 50°$, $c = 20°$. Find the three solid angles A, B, C.

On a piece of tracing paper held in position over the cosine chart (plate 1, also figs. 1 and 3) pass a straight line or simply a straight edge through $b + c = 70°$ (left side of chart) and $b - c = 30°$ (right side of chart) and read the abscissa $A = 60°$ for the point of intersection of the ordinate $a = 42°42'$ with the straight line.

Having found A, the observer reads off the remaining angles $B = 102°$, $C = 26°$, directly from the sine chart (plate 1 and fig. 4) from the relations:

$$\frac{\sin C}{\sin c} = \frac{\sin A}{\sin a} \text{ and } \frac{\sin B}{\sin b} = \frac{\sin A}{\sin a}$$

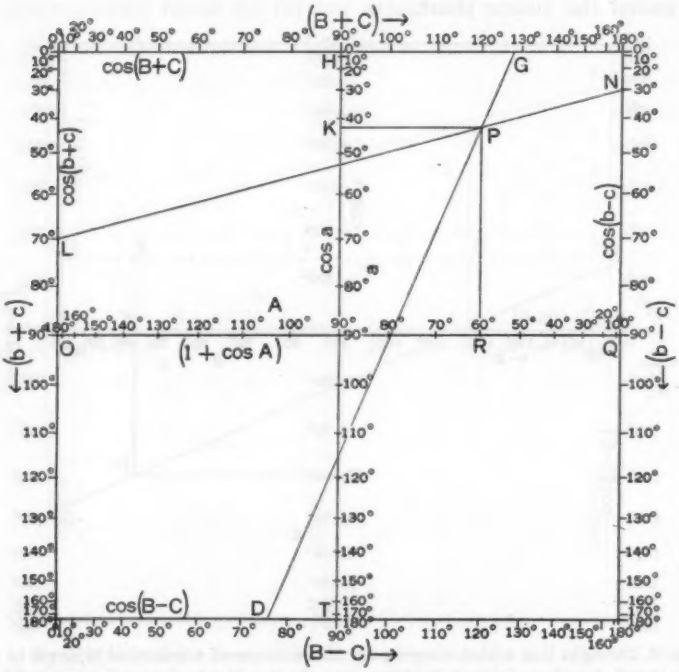


Fig. 3. Straight line cosine diagram for the solution of any spherical triangle in which any three of the six elements are given. In this chart the scales on the horizontal lines refer to the angles A, B, C; the scales on the vertical lines refer to the sides a, b, c. In case two sides and the included angle or the third side are given the construction described under fig. 1 is used; if two angles and the included side or the third angle are given, the construction of fig. 2 is followed except that in this diagram the axes of fig. 2 have been rotated through 90°, as indicated by the labeled scales. For the case that two sides and an opposite angle or two angles and an opposite side are given, it is necessary first to find the second opposite angle of the second side by the use of the sine chart (fig. 4) and to plot the lines LN and DG, the intersection P of which determines the remaining angle and side simultaneously.

It should be noted that when the sine chart is used ambiguity is apt to arise because $\sin (180° - a) = \sin a$. In general this is not serious and the observer can tell on inspection whether the angle given on the sine chart or its supplement is to be taken. In any

spherical triangle $A + B + C > 180°$. In case of doubt the correctness of the angle selected can be tested on the chart by the method followed in examples 5 and 6 below.

(2) Given the three solid angles of a triangle: $A = 60°$, $B = 101°56'$, $C = 25°54'$. To find the three sides a, b, c.

The straight line through the points $(B + C) = 127°50'$ at the top of the cosine chart and $B - C = 76°02'$ at its base intersects the ordinate erected at the abscissa $A = 60°$ at the point $a = 42.7°$ (Plate 1 and figs. 1 and 3). The remaining angles $b = 50°$, $c = 20°$ can be read off directly from the sine chart after A has been found.

(3) Given two sides and the included angle: $b = 50°$, $c = 20°$, $A = 60°$. To find a, B, C.

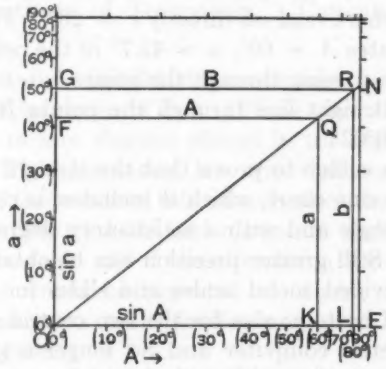


Fig. 4. Straight line sine diagram for the solution of the sine formula of spherical trigonometry. This chart, which has long been used, is the northeast quadrant of plate I and is based on the relation $\sin a : \sin b = \sin A : \sin B$. In fig. 4 $QK = \sin a$, $OK = \sin A$, $RL, = \sin b$, $OL = \sin B$. If any three of these four elements are known, the fourth can be read off directly from the diagram by virtue of the similar triangles OQK and ORL.

Find on the cosine chart the intersection of the straight line passing through the points $b + c = 70°$ and $b - c = 30°$ with the ordinate erected on the abscissa $A = 60°$. The point of intersection is at $a = 42.7°$. The angles $B = 102°$ and $C = 26°$ can now be read off directly from the sine chart. Note that $B = 102°$ and not $B = 78°$ because $A + B + C > 180°$.

(4) Given two solid angles and the included side: $B = 101°56'$, $C = 25°54'$, $a = 42°42'$. To find A, b, c.

Pass a straight edge through the points $B + C = 127°50'$ at top of chart and $B - C = 76°02'$ at bottom of chart and note that it intersects the ordinate $a = 42°42'$ at the abscissa distance $A = 60°$. Read now angles $b = 50°$, $c = 20°$ directly from the sine chart.

(5) Given two sides and an adjacent solid angle: $b = 50°$, $c = 20°$, $B = 101°56'$. To find A, C, a.

From sine chart read angle $C = 26°$. Draw on tracing paper over cosine chart the two straight lines passing through the points $b + c = 70°$ and $b - c = 30°$; and $B + C = 127°56'$, and $B - C = 75°56'$, respectively. Coordinates of the point of intersection of these two lines are the desired angles $A = 60°$ and $a = 42.7°$. In case there is doubt about the value of C, whether 26° or its supplement, 154°, is to be taken, solve for both values and find that for the second set a solution is not possible because the lines do not intersect.

(6) Given two angles and an adjacent side: $B = 101°56'$, $C = 25°54'$, $b = 50°$. To find c, a, A.

From the sine chart read off directly $c = 20°$. Find on the cosine chart the coördinates $A = 60°$, $a = 42.7°$ of the point of intersection of the straight line passing through the points $b + c = 70°$ and $b - c = 30°$ with the straight line through the points $B + C = 127°50'$ and $B - C = 76°02'$.

These examples suffice to prove that the straight line cosine chart together with the sine chart, which it includes, is competent to solve any spherical triangle and with a satisfactory degree of accuracy for many purposes. Still greater precision can be obtained by substituting accurately divided metal scales and slides for the scales at the sides and top and bottom; also for the two central scales; but this is then a trigonometric computer and no longer a graphical method.

In case the degree of precision of plate 1 is not adequate a somewhat higher degree of exactness can be obtained by the use of Chart No. 2776 of the U. S. Hydrographic Office, but relabeled to correspond with plate 1.

SUMMARY

The straight line cosine chart first proposed by M. d'Ocagne for the solution of the triangle for the spherical distance between two points, whose latitudes and longitudes are known, has recently been applied by Littlehales to the solution of the spherical triangle required by the Sumner line method of navigation. Satisfactory as the chart is for this particular problem, its range of usefulness can be extended so as to include the solution of any spherical triangle for which any three angles are known. To attain this result it is necessary to relabel the scales of the d'Ocagne chart and to assign to each of the six identical scales definite angles of the spherical triangle.

A straight line is passed through points on the scales at the sides or the top and bottom of the chart. The unknown angles are then determined by the intersection of this straight line either with a given ordinate or with a second straight line. The accuracy attainable by a chart of this kind is commonly one to two tenths of a degree of arc, depending somewhat on the angles of the spherical triangle; it may mount to a degree or more in an unfavorable case.

The possibility is also suggested of constructing a simple trigonometric computer based on this principle.

SPECTROSCOPY.—*Regularities in the arc spectrum of cobalt.* FRANCIS M. WALTERS, JR., Bureau of Metallurgical Research, The Carnegie Institute of Technology. (Communicated by W. F. MEGGERS.)

Last year a note was published[1] announcing the discovery of regularities in the arc spectrum of iron. These were the first regularities in the spectrum of any element placed in the eighth column of the periodic table. The alternation law was thus verified for the first member of one of the triads in this column since the iron-arc spectrum was found to contain terms of odd maximum multiplicities. Cobalt follows iron in the table and if the alternation law continues to hold, its arc spectrum would be expected to consist of combinations of terms whose maximum multiplicities are even. Such has been found to be the case.

This preliminary note gives twelve multiplets in the quartet system of the arc spectrum of cobalt. These multiplets result from the combination of two P-levels, two D-levels, three F-levels and one G-level, and involve 88 of the 315 lines of cobalt of temperature classes I and II.

Table 1 containing the multiplets shows the types of combining terms, the wave-lengths[2] of the lines involved, the vacuum wave-numbers, the temperature classification,[3] and the intensity, and character in the arc as given by King.[3]

That the levels F and F′ are the lowest of the levels concerned in these multiplets is shown by combinations of the remaining levels with others (not given in this paper) which involve lines of tempera-

[1] This JOURNAL **13**: 243. 1923. Extended results appear in J. O. S. A. & R. S. I. **8**: 245. 1924.

[2] Dhein, Zeitschr. wiss. Phot. **19**: 289–335. 1920.

[3] Astrophys. Journ. **42**: 344–364. Ibid, **51**: 179–186. 1920.

TABLE 1.—MULTIPLETS IN THE ARC SPECTRUM OF COBALT (QUARTET SYSTEM)

Multiplet 1.

	F_5	F_4	F_3	F_2
	80R–II	30r–I	15–I	
	3412.636	3510.419	3584.796	
D_4	29294.50—*815.98*—	28478.52—*590.85*—	27877.67	
		654.28	*654.15*	
		50r–II	20r–I	8–I
		3431.579	3502.63	3552.710
D_3		29132.80—*590.98*—	28541.82—*402.37*—	28139.45
			494.97	*494.82*
			20r–II	15–I
			3442.924	3491.324
D_2			29036.79—*402.52*—	28634.27
				299.05
				25r–I
				3455.236
D_1				28933.32

Multiplet 2.

	F_5	F_4	F_3	F_2
	10–II	4–I	5–I	
	3121.414	3203.030	3264.842	
D'_4	32027.51—*815.86*—	31211.45—*590.88*—	30620.57	
		627.07	*627.02*	
		12–II	4–I	
		3139.943	3199.325	
D'_3		31838.52—*590.93*—	31247.59—	—30845.12
			406.30	
			10–II	5–I
			31493.04	3189.756
D'_2			31743.89—*402.55*—	31341.34
				298.44
				10–II
				3159.660
D'_1				31639.78

Multiplet 3.

	F_5	F_4	F_3	F_2
	100R–II	20r–I		
	3526.856	3631.390		
F''_5	28345.79—*815.93*—	27529.86		
	431.10	*431.40*		
	100R–II	60r–II	15–I	
	3474.019	3575.361	3652.544	
F''_4	28776.89—*815.63*—	27961.26—*590.85*—	27370.41	
		439.04	*439.12*	
		15–II	50R–II	12–I
		3520.087	3594.869	3647.663
F''_3		28400.30—*590.77*—	27809.53—*402.49*—	27407.04
			346.71	*346.81*
			20r–I	40R–II
			3550.599	3602.081
F''_2			28156.24—*402.39*—	27753.85

Multiplet 4.

	F_5	F_4	F_3	F_2
	100R–II			
	3465.796			
G_6	28845.17			
	424.53			
	6–I	50R–II		
	3415.527	3513.483		
G_5	29269.70—*816.02*—	28453.68		
		465.42		
		9–I	30r–I	
		3456.936	3529.037	
G_4	29735.03	28919.10—*590.83*—	28328.27	
				25r–I
				3533.363
G_3		29286.95	28696.12	28293.60

Multiplet 5.

	F'_5	F'_4	F'_3	F'_2
	60–II	10–I		
	3873.117	3974.632		
D_4	25811.70—*659.86*—	25151.84—	—24604.47	
		654.26		
		40–II	15–II	5–I
		3873.907	3957.935	4019.300*
D_3		25806.10—*547.54*—	25258.56—*385.61*—	24872.95
			494.70	*494.84*
			25–I	12–I
			3881.911	3940.895
D_2			25753.26—*385.47*—	25367.79
				299.03
				60–II
				3894.981
D_1				25666.82

Multiplet 6.

	F'_5	F'_4	F'_3	F'_2
	100R–II	25R–I	7–I	
	3502.281	3585.159	3656.965	
D'_4	28544.69—*659.84*—	27884.85—*547.53*—	27337.32	
		627.00	*627.05*	
		80R–II	25r–I	8–I
		3506.315	3574.964	3624.955
D'_3		28511.85—*547.48*—	27964.37—*385.64*—	27578.73
			496.11	*496.11*
			60R–II	20r–I
			3512.643	3560.896
D'_2			28460.48—*385.64*—	28074.84
				298.45
				25r–I
				3523.438
D'_1				28373.29

* This line is also used in multiplet 7.

Multiplet 7.

	F′₅	F′₄	F′₃	F′₂
	20–I	1u		
	4020.904	4130.538		
F″₅	24863.02—*659.90*—24203.12			
	431.39	*431.49*		
	8–I	8–I	3u–IIA	
	3952.329	4058.188	4150.442	
F″₄	25294.41—*659.80*—24634.61—*547.56*—24087.05			
		439.08	*439.10*	
		6–I	4–IA	
		3987.121	4076.134	
F″₃		25073.69—*547.54*—24526.15		—24140.50
			346.80	
			5–I.	3–IA
			4019.300	4082.606
F″₂			24872.95—*385.67*—24487.28	

Multiplet 8.

	F′₅	F′₄	F′₃	F′₂
	20–II			
	3941.735			
G₆	25362.39			
	424.53			
	20–I	10–I		
	3876.840	3978.656		
G₅	25786.92—*659.88*—25127.04			
	465.43	*465.44*		
	10–I	10–I	6–I	
	3808.106	3906.296	3991.693	
G₄	26252.35—*659.87*—25592.48—*547.50*		25044.98	
		367.81	*367.78*	
		4–IA	6–I	6–I
		3850.29	3933.921	3994.541
G₃		25960.29—*547.53*—25412.76—*385.64*—25027.12		

Multiplet 9.

	P₃	P₂	P₁
	80–I		
	6450.231		
D₄	15499.05		
	654.24		
	10–IIA	40–I	
	6188.980	6282.640	
D₃	16153.29—*240.81*—15912.48		
	494.88	*494.90*	
	3	10–I	5–III
	6005.008	6093.138	6231.050
D₂	16648.17—*240.79*—16407.38—*363.15*—16044.23		
		298.72	*299.19*
		3	8–I
		5984.182	6116.982
D₁		16706.10—*362.68*—16343.42	

Multiplet 10.

	P'_3	P'_2	P'_1
	15–I		
	7084.970		
D_4	14110.50		
	654.18		
	20–I	8–I	
	6771.05	7052.854	
D_3	14764.68—*589.92*—14174.76		
	494.80	*494.82*	
	3–III	15–I	3–I
	6551.45	6814.954	7016.596
D_2	15259.58—*590.00*—14669.58—*421.57*—14248.01		
		299.01	*298.99*
		4–II	10–I
		6678.812	6872.38
D_1		14968.59—*421.59*—14547.00	

Multiplet 11.

	P_3	P_2	P_1
	40–I		
	5483.336		
D'_4	18232.02		
	626.99		
	15–II	20–I	
	5301.036	5369.580	
D'_3	18859.01—*240.74*—18618.27		
	496.14	*496.13*	
	3–IV?	25–II	15–II
	5165.148	5230.204	5331.450
D'_2	19355.15—*240.75*—19114.40—*362.98*—18751.42		
		298.44	*298.50*
		4–II	15–II
		5149.800	5237.912
D'_1		19412.84—*362.92*—19049.92	

Multiplet 12.

	P'_3	P'_2	P'_1
	5–III		
	5935.372		
D'_4	16843.48		
		1u	
		5922.350	
D'_3	17470.47	16880.53	
D'_2	17966.69	17376.73	16955.12
D'_1		17675.10	17253.49

ture classes III, IV, and V. It is noteworthy that six of the eight multiple levels occur in pairs separated by rather small wave-number intervals: $P' - P = 1300$; $D' - D = 2700$; $F' - F = 3800$.

In the region covered by the measurements of Dhein, de Gramont has indicated seven lines as *raies ultimes*, of which one, 3474.019A, appears in the multiplets given here. The "persistent" lines observed by Pollok and Leonard,[4] 3412.636, 3465.796, 3474.019, 5302.281 and 3873.117A are found one to a multiplet.

Seven of the lines for which Rybar[5] gives the Zeeman effects are found in these multiplets. A comparison of the patterns observed by Rybar and those calculated by Landé's scheme[6] is given in Table 2.

TABLE 2.—ZEEMAN EFFECT FOR COBALT LINES

λ	TERMS	CALCULATED	OBSERVED
3455.236	D_1F_2	(0.20) 0.20, **0.60**	(0.20) 0.59
3523.438	D_1F_3	(0.20) 0.20, **0.60**	(0.20) 0.59
3894.981	D_1F_3	(0.20) 0.20, **0.60**	(0.20) 0.60
3491.324	D_2F_2	(0.40, **1.20**) 0.00, **0.80**, 1.60	(1.18) **0.02**, 0.80, 1.59
3560.898	D_2F_3	(0.40, **1.20**) 0.00, **0.80**, 1.60	(1.21) **0.00**, 0.80, 1.59
3940.895	D_2F_2	(0.40, **1.20**) 0.00, **0.80**, 1.60	(1.17) **0.00**, 0.79, 1.58
3585.159	D_4F_4	(0.10, 0.27, 0.48, **0.67**) 0.76, 0.95, 1.14, **1.33**, 1.52, 1.71, 1.90	(0.51) 1.36

It is of interest to compare the structures of the arc spectrum of cobalt with those of the preceding element, iron. Whereas the terms found in the arc spectrum of iron belong to systems of odd permanent multiplicity (triplets, quintets, septets), those found for cobalt belong to even systems. In addition to the quartet system represented by the above multiplets there is good evidence of doublet terms, and terms belonging to the sextet system are expected in the final analysis.

In the arc spectrum of iron all of the terms are "inverted," that is, in a given multiple level the magnitude of the term increases with the inner quantum number, whereas the terms in cobalt are normal, resembling those of the majority of elements with complex spectra. This abrupt change from iron is of interest, since in approaching iron along a row of the periodic table from the other direction we find titanium and vanadium with all terms normal, chromium with a few inverted terms, manganese with many and iron with all terms inverted.

The separations of the sub-levels of the quartet system of cobalt average 3 to 4 times larger than corresponding separations for the sub-levels of the quintet-system terms in iron.

[4] Roy. Dublin Soc. Proc. **11**: no. 18. 1907.
[5] Physik. Zeitschr. **12**: 889. 1911.
[6] Zeitschr. für Physik. **15**: 190. 1923.

BOTANY.—*New plants from the Dominican Republic.*[1] E. C. LEONARD, National Museum (Communicated by WILLIAM R. MAXON).

The new species described in this paper are based upon material collected by Dr. W. L. Abbott in the early part of 1922. Prior to 1920 only a small proportion of the plants known to occur in the Dominican Republic were represented in the National Herbarium by specimens from that country, these consisting mainly of collections by Wright, Parry, and Brummel in 1871, Rose in 1913, and certain numbers of Türckheim, Fuertes, and Raunkiaer received from Berlin and Copenhagen. During the past four years, however, Dr. Abbott has collected nearly 3,000 numbers. Of these many have proved new to science and a considerable number are new to the National Herbarium.

Botanically, Hispaniola is probably the least known of the West Indies, and both Haiti and the Dominican Republic offer an extremely rich field to the collector.

Sophora albo-petiolulata Leonard, sp. nov.

Plant shrubby; stipules none; leaves 5 to 10 cm. long, the rachis white-pubescent, deeply grooved on upper side; leaflets 12 to 16, elliptic, 8 to 10 mm. long, 2 to 5 mm. wide, rounded at both ends, entire, recurved on margins, obscurely punctate, the upper surface bright green, the lower surface paler; petiolules yellowish white, about 1.5 mm. long, pubescent; racemes terminal, 8 to 10 cm. long, velvety-pubescent; flowers 14 to 20, white, the standard broadly ovate, abruptly narrowed to a claw, 15 mm. long, 13 mm. broad, emarginate, the other petals 12 to 13 mm. long, 4 to 5 mm. broad; stamens 10, 9 to 10 mm. long; calyx 5 mm. long, silky-pubescent; pedicels 6 to 7 mm. long, silky-pubescent; ovary densely white-pubescent; pods not seen.

Type in the U. S. National Herbarium, no. 1,079,459, collected at Maniel Viejo, Province of Barahona, Dominican Republic, altitude 750 meters, March 10, 1922, by W. L. Abbott (no. 1934). Abbott's no. 1911 collected at the same locality is of this species.

No West Indian *Sophora* closely related to this species has hitherto been described. In shape and size of the leaflets it bears some resemblance to *S. macrocarpa* from Chile, but differs in its much smaller flowers.

Zanthoxylum bifoliolatum Leonard, sp. nov.

Tree, glabrous throughout, the branches sparingly armed with straight spines 2 to 4 mm. long; petioles 2 cm. long, grooved on upper surface, bearing a pair of spines at base of leaflets; leaflets 2, opposite, obovate, 4 to 8 cm. long, 2 to 4.5 cm. wide, rounded and emarginate at apex, gradually narrowed to a subsessile, often oblique base, coriaceous, entire, the upper sur-

[1] Published by permission of the Secretary of the Smithsonian Institution.

face shining, prominently veined, the lower surface duller, minutely glandular, the veins prominent, the midrib usually bearing a spine below the middle; flowers 2 to 3 in the axils of the leaves; staminate flowers on pedicels 2 to 2.5 mm. long; calyx 3-lobed, minute; petals 3, ovate, 2 mm. long, 1.5 mm. broad; stamens 3, 4 to 5 mm. long; fruit not seen.

Type in the U. S. National Herbarium, no. 1,079,511, collected on Quita Espuela, in the vicinity of San Francisco de Macorís, Provincia Pacificador, Dominican Republic, altitude 1,000 meters, April 7, 1922, by W. L. Abbott (no. 2083).

Zanthoxylum bifoliolatum differs from other tropical American plants of this genus in having only two leaflets and in its small axillary inflorescence.

Zanthoxylum venosum Leonard, sp. nov.

Tree (?) or shrub, glabrous throughout, armed with straight conical spines 2 mm. long; leaves odd-pinnate, 6 to 12 cm. long; petioles seldom armed, not grooved; rachis sparingly armed with short spines, shallowly grooved above; leaflets usually 5, opposite, elliptic to obovate, 3 to 7 cm. long, 1.5 to 3 cm. wide, narrowed to an obtuse point at apex, narrowed at base to a grooved petiolule 3 to 6 mm. long, prominently nerved and veined, very firm, coriaceous, entire, glabrous, both surfaces pellucid-glandular, shining, dark green above, paler beneath, the midrib on the under surface often bearing a small spine near the middle; inflorescence terminal, corymbose, glandular, armed with slender spines 2 to 2.5 mm. long, the flowers numerous; staminate flowers on pedicels 1.5 to 2 mm. long; sepals 3, 1 mm. long, ovate, rounded at apex; petals 3, ovate, 2.5 mm. long, 1.5 mm. wide; stamens 3, equaling the petals; pistillate flowers and fruit not seen.

Type in the U. S. National Herbarium, no. 1,079,475, collected on the summit of Loma de Cielo, near Polo, Province of Barahona, Dominican Republic, altitude 1,300 meters, March 12, 1922, by W. L. Abbott (no. 1969).

Although apparently related to *Zanthoxylum coriaceum* A. Rich., this plant differs strongly from that species in its more pointed, conspicuously veined, pellucid-glandular leaves and armed stems.

Maba urbaniana Leonard, sp. nov.

Shrub or tree (?); bark gray, the tips of the young branches densely brown-pubescent; petioles 2 to 3 mm. long, densely pubescent; leaves obovate to oval, 1.5 to 3 cm. long, 0.5 to 2.5 cm. wide, rounded and occasionally emarginate at apex, narrowed from above the middle to a truncate or subcordate or rarely rounded base, entire, strongly reticulate, the upper surface shining, glabrous or slightly pubescent, drying tawny-olive, the veins impressed, the lower surface densely and softly pubescent, drying buckthorn-brown, flowers 2 to 3 in short axillary clusters; pedicels 2 to 3 mm. long, densely and softly tawney-brown pubescent; calyx 3 mm. long, 3-lobed, the lobes broadly triangular, pubescent without, glabrous within; corolla about 8 mm. long, pubescent, 3-lobed, the lobes 1 mm. long, thick, leathery; stamens 10, the longer 3 mm., the shorter 1.5 mm.; fruit not seen.

Type in the U. S. National Herbarium, no. 1,179,585, collected in the vicinity of San Lorenzo Bay, Dominican Republic, near sea level, April 26, 1922, by W. L. Abbott (no. 2235).

This species is distinct from its nearest relative, *M. caribaea* (A.DC.) Hieron, in the smaller and more crowded leaves and the denser soft brown pubescence of the under surface of the leaf blades.

Solanum abbottii Leonard, sp. nov.

Shrub, 1 to 1.5 meters high; young twigs rather densely stellate-pubescent with 5 to 7-rayed hairs and armed with straight yellow spines about 5 mm. long; petioles 10 to 15 mm. long, stellate-pubescent and spinose; leaves oblong-elliptic in outline, 10 to 15 cm. long, 3 to 6 cm. wide, acutish at apex, narrowed from near the middle to base, sinuately 5 to 7-lobed, the lobes obtuse, the sinuses shallowly rounded, the upper surface dull green with occasional spines and stellate hairs on the impressed veins, otherwise glabrous, the lower surface yellowish green, rather densely pubescent with easily detached stellate hairs and with occasional spines; racemes axillary, up to 8 cm. long, 8 to 14-flowered; pedicels 6 to 10 mm. long, glabrous; calyx stellate-pubescent and spiny, the lobes triangular, abruptly long-acuminate; corolla violet, the lobes lance-ovate, 10 to 12 mm. long, 4 mm. broad, acutish, glabrous within, stellate-pubescent without; stamens and style equal and nearly erect, the anthers 4 to 5 mm. long; fruit not seen.

Type in the U. S. National Herbarium, no. 1,079,509, collected on the summit of Quita Espuela in the vicinity of San Francisco de Macorís, Provincia Pacificador, Dominican Republic, altitude 900 meters, April 7, 1922, by W. L. Abbott (no. 2073). An additional specimen was collected in the same region, April 10, 1922, by W. L. Abbott (no. 2124).

According to description, *Solanum orthacanthum* O. E. Schulz[2] agrees with the proposed new species in many respects, especially in general leaf-shape and in the character of the pubescence, but differs in having larger leaves (15 to 20 cm. long) with oblique cordate base and faintly repand margin, fewer flowers (about 4 in each raceme), shorter tomentose pedicels (2 to 4 mm. long), and larger corolla lobes (lobes 15 mm. long).

Tabebuia rugosa Leonard, sp. nov.

Small tree; young branches thick, gray; leaves 3-foliolate; petioles very stout, 1 cm. long; petiolules 1 cm. long or less; leaflets ovate to elliptic, 12 to 17 cm. long, 6 to 11 cm. wide, rounded and usually minutely apiculate at apex, obliquely subcordate at base, coriaceous, the margins entire or undulate, recurved, the upper surface glabrous, lepidote (?), strongly rugose and shining, the veins impressed, the lower surface drying light grayish brown, prominently and reticulately veined, the midrib and primary and secondary veins glabrous, the anastomosing veins densely and softly pubescent, otherwise glabrous; inflorescence terminal, umbellate, brown-lepidote; pedicels 4 to 5 mm. long; calyx 12 to 13 mm. long, lepidote, the lobes triangular, acute; corolla 3 to 3.5 cm. long, "red," the stamens inserted on the corolla tube 5 mm. above the base, the longer stamens 15 mm. long, the shorter 12 mm., the anthers 3.5 mm. long.

Type in the U. S. National Herbarium, no. 1,079,609, collected near Lajana on the Samaná Peninsula, Dominican Republic, altitude 100 meters, April 30, 1922, by W. L. Abbott (no. 2272).

This species may be related to *T. acrophylla* (Urban) Britton, as a similarity in texture and shape of leaves seems to indicate, but in other respects

[2] Urban, Symb. Antill. 7: 537. 1903.

the two plants are different. The flowers of *T. acrophylla* are twice as large and the leaflets are smaller and obovate and pubescent on both surfaces, including the primary and secondary veins of the lower surface.

Dr. Abbott's no. 2293, collected in the same region, May 2, 1922, and possibly from the same tree, consists of leaves and mature pods. The pods are up to 18 cm. long, 5 mm. thick, rough-lepidote, with slender awn-like tips; the seeds are 7 mm. long, 5 mm. wide, the wings 5 to 7 mm. long.

Tabebuia rubriflora Leonard, sp. nov.

Large tree; twigs thick, gray; leaves 4 or 5-foliolate, the petioles up to 8 cm. long; petiolules 0.5 to 2 cm. long; leaflets elliptic to obovate-oblong, 5 to 12 cm. long, 2.5 to 6 cm. wide, obtusish at apex, rounded or subcordate at base, coriaceous, entire, slightly lepidote on both surfaces, pubescent in the angles of the veins on the lower surface, otherwise glabrous; flowers in axillary, 2 to 4-flowered, subsessile clusters; pedicels 8 to 10 mm. long; calyx 6 to 7 mm. long, sparsely lepidote, the lobes triangular, 2 to 3 mm. long, acute; corolla 2.5 to 3 cm. long, deep red, the tube 2.5 mm. wide at base, 15 mm. wide at throat; capsules smooth, up to 12 cm. long; seeds 10 mm. long, 4 to 5 mm. wide, the wings 4 to 5 mm. long.

Type in the U. S. National Herbarium, no. 1,079,657, collected in the vicinity of Laguna, on the Pilón de Azúcar, Dominican Republic, altitude 500 meters, May 12, 1922, by W. L. Abbott (no. 2346).

In texture and size of leaflets this species resembles *T. pachyphylla* Britton, from Cuba, but differs in its much smaller flowers and broader leaflets.

Tabebuia paniculata Leonard, sp. nov.

Shrub or small tree; petioles 1 to 2 cm. long; leaves 3 to 5-foliolate, the petiolule of the middle leaflet 5 to 10 mm. long, the lateral leaflets subsessile; leaflets elliptic to obovate, 3 to 8 cm. long, 1.5 to 4 cm. wide, rounded and emarginate at apex, gradually narrowed at base (the lateral leaflets oblique), thick, coriaceous, entire, the upper surface with impressed nerves, glabrous or slightly lepidote, drying dark brown, the lower surface with prominent nerves, lepidote, drying light brown; inflorescence terminal, paniculate, lepidote, the bracts subulate 1.5 to 8 mm. long, the flowers numerous; pedicels 1 to 2 cm. long, lepidote; calyx 2-lipped, 10 to 12 mm. long, lepidote; corolla 4 cm. long, purplish (?), veiny, the tube 3 mm. wide at base, enlarging to 15 mm. at throat; pods not seen.

Type in the U. S. National Herbarium, no. 1,079,633, collected in the vicinity of Laguna, on the Pilón de Azúcar, Dominican Republic, altitude 500 meters, May 11, 1922, by W. L. Abbott (no. 2330).

Tabebuia paniculata is related to *T. pentaphylla* Hemsl., and differs chiefly in its short petioles and more rounded leaflets. It resembles *T. gonavensis* Urban in some respects, but has much larger leaflets and smaller flowers.

Justicia alsinoides Leonard, sp. nov.

Diffuse herb; stems weak, branched, usually decumbent, rooting at the nodes, 10 to 15 cm. long, terete, obscurely grooved, pubescent with downwardly curved hairs, bearing white linear straight cystoliths 0.25 mm. long; petioles 5 to 8 mm. long, the under surface pubescent with curved hairs,

the upper bearing cystoliths; leaves ovate, 10 to 18 mm. long, 8 to 10 mm. wide (the upper and lowermost usually smaller), obtuse at apex, rounded, subcordate, or abruptly narrowed at base, shortly decurrent on petiole, the upper surface bearing numerous cystoliths, glabrous or with a few scattered hairs, the lower surface glabrous, bearing a few cystoliths; inflorescence spicate, 2 to 4 cm. long, 4 to 6-flowered; bracts linear, lanceolate, 2.5 mm. long; calyx lobes linear-lanceolate, 4 mm. long, sparsely pubescent, 3-nerved, the lateral nerves less prominent than the median; corolla 5 mm. long, pinkish, the tube glabrous, the lips equal, 2 mm. long, puberulent without, the upper ovate, notched, the three lobes of the lower lip equal in size, rounded; style 3.5 mm. long (?); capsule 6 mm. long, puberulent, 4-seeded, the retinacula 1 mm. long; seeds flat, stipitate at base, orbicular, 1 mm. in diameter, reddish brown, minutely reticulate.

Type in the U. S. National Herbarium, no. 1,979,513, collected on the summit of Quita Espuela, in the vicinity of San Francisco de Macorís, Provincia Pacificador, Dominican Republic, altitude 900 meters, April 8, 1922, by W. L. Abbott (no 2119a).

This plant is closely related to *J. reptans*, and may be only a form of that species. It differs chiefly in the size of the sepals, which are at least twice as long as those of *J. reptans*. Likewise, both corolla and capsule are noticeably larger and the leaves are more rounded at the base.

Siphocampylos linearifolius Leonard, sp. nov.

Herbaceous; stems 36 cm. high (or higher?), glabrous; leaves narrowly ovate-oblong, 4 to 7 cm. long, 4 to 11 mm. wide, tapering from below the middle to an obtuse apex, narrowed to a short winged petiole 1 to 2 mm. long, serrate with small sharp teeth or the upper nearly entire, glabrous on both surfaces; flowers in the axils of the upper leaves; pedicels 2 cm. long, puberulent, bearing two subulate bracts near the middle; calyx lobes narrowly triangular, 6 to 7 mm. long, 1 mm. wide at base, puberulent, often reddish; corolla 2.5 to 3 cm. long, red, the lips equal, the lobes of the lower lip narrowly triangular, 5 mm. long, 2 mm. broad at the base, obtuse at apex, yellow in dried plants, the lobes of the upper lip ovate, 7 to 8 mm. long, 3 mm. broad at base, obtuse at apex; styles 2 cm. long; anthers 3.5 to 4 mm. long, densely barbellate at apex; capsule 6 mm. long; seeds 0.5 mm. in diameter, grayish brown, reticulate.

Type in the U. S. National Herbarium, no 1,079,436, collected at Maniel Viejo, Province of Barahona, Dominican Republic, altitude 1060 meters, March 8, 1922, by W. L. Abbott (no. 1903). Dr. Abbott's no. 1851, collected in the same region from the upper slopes of Loma de Cielo, near Polo, altitude 1,200 meters, March 2, 1922, is of this species.

Related to *S. tuerckheimii* Urban in the similarity of the flowers, the glabrous stems, and thin glabrous leaves, this plant is sufficiently distinct in its very narrow leaves and more distant teeth to deserve specific rank.

SCIENTIFIC NOTES AND NEWS

W. TAYLOR THOM, JR., has been appointed geologist in charge of the newly formed section of Geology of Fuels in the Division of Geology, U. S. Geological Survey.

E. M. SPIEKER, geologist, has been granted leave of absence from the U. S. Geological Survey to give a course of instruction in geology at Ohio State University.

MAXWELL N. SHORT was appointed junior scientist in the U. S. Geological Survey and has been assigned to work in Section of Petrology.

DR. JOSEPH F. ROCK sailed from San Francisco on September 30 for China, as head of an expedition that is to conduct botanical exploration in the interior of that country for the Arnold Arboretum. Dr. Rock returned early in the summer from a similar expedition, in the interest of the National Geographic Society and the Department of Agriculture, to Burma, Yunnan, and the borderland of Tibet where he collected approximately 70,000 specimens of plants. Work on this large collection is now in progress at the National Museum.

Miss CLARA SOUTHMAYD LUDLOW, entomologist at the Army Medical Museum, died September 29. Her work was mainly in connection with the disease prevention activities of the army.

XEROX